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Recently a protocol for Quantum Clock Synchronization (QCS) of remote clocks using quantum entanglement was proposed by Jozsa *et al.* This method has the goal of eliminating the random noise present in classical synchronization techniques. However, as stated QCS depends on the two members of each entangled pair undergoing the same unitary evolution even while being transported to different locations. This is essentially equivalent to a perfect Eddington Slow Clock Transfer protocol and thus, not an improvement over classical techniques. We will discuss this and suggest ways in which QCS may still be used.

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In a preprint entitled, "Quantum Clock Synchronization Based on Shared Prior Entanglement" (hereafter referred to as QCSP) recently posted to the LANL quantum physics archive [1], the authors outline a technique for using quantum non-locality to synchronize remote clocks without the usual errors inherent in classical time transfer techniques. It is our view that, as stated, this scheme has several fundamental limitations that will prevent it from achieving clock synchronization better than can be accomplished using classical techniques. It is worth noting at the outset that our objections are not based on the fact that these limitations are hard to overcome, but that they are inherently the same limitations found in classical synchronization methods. To overcome them means that the problem is already solved classically, without resorting to quantum mechanical methods.

While the possibility of using quantum non-locality to synchronize clocks is intriguing, it would appear that any such scheme must be independent of *how* information is transmitted classically. In the case of the protocol outlined in QCSP, information is being transmitted classically in two ways. First, through the usual classical channel that accompanies entanglement experiments and second, the relative phase between two partners in an entangled pair is transmitted with the entangled particles themselves as they are transported from one site to another. Since this phase will necessarily be shifted by the classical transportation process, such a dependence on classical conveyance of this information will make this QCS protocol equivalent to existing classical methods such as the Eddington Slow Clock Transport (ESCT) protocol [2].

In ESCT two local clocks are synchronized and one of them is moved "slowly" to a remote site where it can remain or where it can transfer the synchronization with the first clock to a third remote clock. "Slowly" is defined as a velocity such that velocity-dependent perturbations to the clock are lower than any other noise in the problem. In clock jargon this is sometimes called a "perfect clock trip". In comparison, protocols for Quantum Cryptography [3], which are similar to QCS, are not sensitive to the relative phase of the entangled particles as they are transmitted.

The authors start with a two-level system consisting of states $|0\rangle$ and $|1\rangle$ and energy eigenvalues E_0 and E_1 with corresponding frequency $\Omega = \frac{1}{\hbar}(E_1 - E_0)$. A dual basis $|pos\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|neg\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ is then defined along with the "Hadamard transform", H, that maps $|0\rangle$ into $|pos\rangle$ and $|1\rangle$ into $|neg\rangle$. The claim is made that two consecutive applications of H separated by time, t, is equivalent to a Ramsey temporal interferometer. While the time evolution of the states after the first application of H in QCSP eq. 1 is essentially correct (up to an overall time-evolving phase), the expression for the probability of measuring the system in one state or the other in eq. 2 is not correct in the context of the Ramsey method as it is applied to atomic clocks [4]. In a Ramsey interferometer with interrogation on resonance, the probability of measuring the system in state $|0\rangle$ or $|1\rangle$ is independent of the time between the two pulses (the "Ramsey time").

The problem lies in the fact that the authors have not included a key part of the Ramsey interferometer: the relative phase between the interrogation oscillator and the precession of the system. Later this phase ambiguity is addressed and we will come back to it as well.

Even though H may not correspond exactly to a Ramsey interferometer, it is likely that it can be realized and that it would produce the desired result (QCSP eq. 2). Let us assume that this is true and that the result of the measurements described is as stated in eq. 2. The authors then state that two separated observers, Alice and Bob (hereafter referred to as A and B), share an ensemble of singlet states of the form, $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$. The statement is made that, "... this singlet state ... does not evolve in time *provided A and B undergo identical unitary evolutions...*" (emphasis added). It is not clear from the paper whether the authors intend these unitary transformations to *include* the transportation of the entangled particles. If transportation *is* included then the statement that A and B undergo identical unitary evolutions is equivalent to the statement that A and B are able to build perfect clocks and transport them perfectly. Thus, they have effectively solved time synchronization completely and with only classical methods thereby making QCS unnecessary.

On the other hand, if they are not including the transportation process in the unitary transformations, then they still have problems. First, they are ignoring the phase shift introduced by transporting one of the particles in an entangled pair. This phase would usually be caused by Doppler shifts and electromagnetic field perturbations. While these can be minimized, the essential point is that they are of the same type whether one is referring to an entangled pair or an atomic clock. Therefore, the authors seem to be implicitly assuming a perfect ESCT. If one has perfect ESCT, then one doesn't need QCS, at least to the extent that transporting a clock is no more difficult than transporting ensembles of entangled particles.

The second problem is that A and B almost certainly do not undergo identical unitary evolutions. To do so would mean that they have perfect control over at least their environments and therefore can build perfectly stable clocks. If they have perfect clocks then they have already achieved frequency synchronization without QCS simply by virtue of having built their respective clocks. Even if one assumes that perfect clocks could be built, the resultant frequency synchronization could not be translated, in general, into time synchronization by QCS because of the transport problems already discussed.

The authors next define the QCS protocol: 1) an ensemble of entangled particles called "pre-clocks" is created in the $|pos\rangle, |neg\rangle$ basis; 2) A starts the clocks by measuring her half of the ensemble in that basis, thereby also determining which members in her ensemble are of type $|pos\rangle$ (type I) and which are of type $|neg\rangle$ (type II); 3) A communicates type I/II information to B over a classical channel; 4) B selects out type II (in phase with A's type I) particles and measures them in the $|pos\rangle, |neg\rangle$ basis to establish the synchronization with A.

At this point the authors note that the protocol is incomplete because $|pos\rangle$ and $|neg\rangle$ are not uniquely determined. In the case of an atomic clock the $\pi/2$ pulse that defines H in that context has a phase relative to the atoms. This is the relative phase between the interrogation oscillator and the precession of the system that prevents QCSP eq. 2 from representing a real atomic clock. Since knowing this phase is equivalent to solving the problem in the first place (as the authors point out), they propose adding a second oscillation frequency and observing the "beat note" between the two frequencies to eliminate the phase ambiguity, δ. However for this to work, δ must be the same for both frequencies. In general this would not be true because it must include the phase shift due to the transport process which will usually be frequency dependent. If the two δ's in QCSP eq. 7 are *not* the same, then the beat envelope is not independent of this additional phase ambiguity and it hasn't been resolved. For example, if the particles are different isotopes of an atom, then their sensitivities to external perturbations will be different leading to a different accumulated phase. To ignore this is equivalent to once again assuming a perfect ESCT. In addition the two accumulated phases (for each species) are likely to be time-dependent once the trip is complete (imperfect clocks).

Since the arbitrary phase δ is not resolved, time synchronization is not achieved. Even if the rest of QCS were to work, the best one could hope for would be frequency synchronization ("syntonization") which is a much easier class of problem. However given our objections, as it stands, QCS probably isn't able to do syntonization without perfect clocks and as has already been pointed out, perfect clocks provide syntonization "for free". On the other hand, there may be ways to still get useful information. For instance, if the clocks belonging to A and B are known to differ only by a constant rate then A and B could double the size of their ensembles and perform two separate QCS protocols. The difference in results would be the phase accumulated *only* while the ensembles were in their remote locations, and not due to the transport process (of course this necessarily includes phase errors introduced by imperfect environmental control, but these will be significantly less than those introduced by the transport process). This procedure, together with the assumption of a linear rate would give the frequency offset of the two clocks. They could further increase the size of their ensembles (all transported at the same time) and make additional QCS measurements at later times to retain the frequency information over longer periods. The two clocks will still not be phase-locked, but important error-free information will have been determined about their respective operation at remote sites.

At first, it may seem that a way around the problems of transporting entangled ensembles over large distances may be overcome by using entangled photons which are sent from A to B. The entanglement is then transferred to the system of choice (e.g., atoms) for storage and carrying out of the protocol. Leaving aside the engineering details of accomplishing this on a global scale, one must know the propagation delay of the photons in order to determine the phase of the entanglement that is transferred. We are assuming that the phase of the photon is written onto the phase of the atom during entanglement transfer which is true in most resonant atom-photon interactions. With this assumption, transferring the entanglement with photons is also still equivalent to the classical problem.

In summary we believe that the QCS protocol is an interesting area of study, but that as stated, it will work no better than existing classical methods. We emphasize that our objections are not based on the existence of hard engineering problems (which there are), but that the problems that must be solved for QCS to work (perfect clock trips) appear to be the same as those for classical methods. However, that is not to say that there isn't some other protocol that will make QCS work.

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